Research on Sustainable Development Based on Entropy-Ge Matrix and Lanchester's Equation

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Abstract: Due to the continuous deterioration of the Earth's environment and rapid depletion of resources, we urgently need to implement sustainable development strategies to meet human survival needs and benefit future generations. Sustainable development should focus on both development and coordination. This article will establish a sustainable development assessment and prediction model, aiming to identify sustainable and unsustainable countries in the world. It can identify the most urgent sustainability issues of a country, help it formulate development plans more accurately, allocate resources reasonably, and increase its sustainability. This article will use the Entropy-GE Matrix and Lanchester's equation to construct a model to evaluate and measure a country's sustainability, and predict sustainability trends. Firstly, a sustainable development indicator system was established to standardize the indicator data. Secondly, the entropy GE matrix is used to construct the Development Index (DI) and Collaboration Index (CI) to measure the final comprehensive performance score. Finally, by establishing the Lanchester's equation applicable to sustainable development, it can not only effectively describe the mutual influence between different levels (economic, social, environmental) within a country, but also take into account the influence of external factors. Obtain the dynamic process of sustainable development and predict changes in sustainability through the Lanchester's equation.

1. Introduction

The theory of Sustainable Development(SD) has evolved over a long period of time. In the 1950s and 1960s, "development = economic growth" model was questioned and studied as a response to the environmental pressures of economic growth, urbanization, population and resources. It was not until 1983 that the World Commission on Environment and Development (WECD) was established, and in 1987 the study "Our Common Future" was published, using "sustainable development" as its basic platform. Sustainable development attempts to establish a relationship in which society, economy, population, resources, and environment are in harmony with each other and develop together, with the aim of meeting the needs of the present generation relatively well, without jeopardizing the development of future generations. Trying to achieve the balance that sustainable development is trying to achieve is particularly difficult. Unfortunately, the difficulty is even greater, especially for developing countries. This paper will develop a model for the sustainable development of a country. This model should provide a measure that distinguishes more sustainable countries and policies from unsustainable ones. In this way, international organizations will be informed of those countries that are most in need of support and intervention.

2. Model

2.1 Assumption

(1) The situation in each country is relatively stable, meaning that the country will not undergo extremely significant changes in a short period of time, such as sudden and devastating natural disasters and wars.

(2) The influence of external factors on the model follows a Gaussian distribution. External

factors hardly have a strong impact on the model, making it easy to quantify external effects in research.

(3) The basic data of a country can be counted, such as economic data, population data, resource data, and so on.

(4) The operating mechanism of a country is relatively independent of other countries. Due to the inherent political and economic structure that largely affects a country's economic factors, it is assumed that the internal operating mechanisms of each country are solely dependent on that country and not influenced by other countries.

(5) The statistical data is valid and reliable. Assuming that the true value of each parameter is located near the statistical data.

(6) The values of each parameter can be calculated from the existing dataset.

2.2 Basic Process

The main purpose of this article is to assess sustainability of a country and predict its future development trends, in order to facilitate the country's formulation of sustainable development strategies that are suitable for itself and receive precise assistance. Firstly, a sustainable development indicator system was created and a sustainability measurement method based on entropy theory and ge matrix was provided. Secondly, use the theories of the Lanchester equation and equation of state to predict sustainability. Finally, the comprehensive performance scores of the three dimensions (economic, social, and environmental) of the country were obtained, and the sustainable development plan was studied and controlled and adjusted in sequence. The specific process is shown in Figure 1 below.

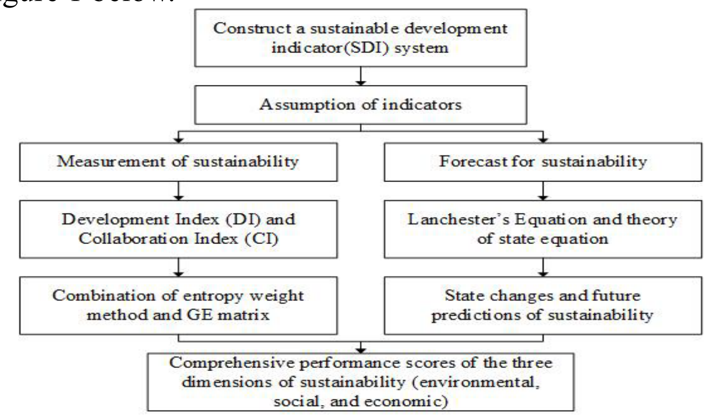

Fig.1 Flow Chart of Basic Process

2.3 Establishment of Sustainable Development Indicators(SDI)

Sustainable development encompasses two important concepts: the concept of needs, especially the basic needs of people in all countries of the world, which should be considered as a special priority, and the concept of limits, the limits imposed by the state of technology and the organization of society on the ability of the environment to meet current and future needs."[1] The development of a country is composed of many complex factors, among which there are three key elements: social, economic, and national. Sustainable development advocates harmonious and comprehensive development, with the environment as the foundation and the economy as the guarantee. The ultimate goal is to achieve sustainable social development. At this stage, it is urgent to achieve sustainable development to improve the status quo. Therefore, establishing appropriate sustainable development indicators is crucial. By studying and combining over 100 indicators identified by the Eurostat[2], with a focus on 12 overall indicators, 52 international indicators constructed by Shen Li to evaluate sustainable urbanization[3], and a case study of 37 indicators from China's SDI study conducted by Jelena Zlatar from 2001 to 2012[4], we have identified the following sustainable development indicator system as Table 1. These indicator systems should meet the following conditions:

- The effectiveness of the indicator should preferably have been tested

•It is best to calculate indicators from existing datasets
•It is best that indicators can be artificially controlled by the government and policies

Table 1 Indicator System Of Sustainable Development

Level 1	Level 2	Level 3
Environmental	Population	Population growth
		Population density
		Aging rate
	Resource	Forest rents in GDP
		Energy use per capita
	Pollution	CO_2 emission per capita
Social	Health	Life expectancy
		Mortality rate
		Health expenditure per capita
	Education	Tertiary school enrollment rate
		Education spend in GDP
	Livelihood	Depth of the food deficit
		Improved water source with access
		Household expenditure per capita
Economic	Per Capita GDP	
	Inflation	
	Tertiary industry rate	

2.4 Assessment of Sustainability

2.4.1 The Entropy Method

There are many methods for calculating weights, each with its own advantages and disadvantages. For example, due to the strong subjectivity of Analytic Hierarchy Process (AHP), the determination of the judgment matrix depends on experts, and subjective judgments have a significant impact on the results. Therefore, the entropy weight method is introduced to calculate weights more objectively.

(1) The various indicators of each group of data can be written as an m × n matrix, where m refers to number of evaluation objects and n refers to number of evaluation indicators. The indicator system matrix is as Equation (1):

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{12} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \tag{1}$$

(2) Normalization of indicators

As shown in the following Table 2, indicators are classified to four categories. The normalization of indicators requires converting all indicators into extremely large ones.

Table 2 Classification of Indicators

Indicator Name	Indicator characteristics
Extremely large indicators	Larger is better
Extremely large indicators	Smaller is better
Intermediate indicators	Closer to a certain value is better
Interval indicators	Falling within a certain range is best

•Extremely small indicators

$$x_i = \max\{x_1, x_2, \dots, x_i\} - x_i \tag{2}$$

If all elements are positive, the reciprocal can be taken directly:

$$x_i = \frac{1}{x_i} \tag{3}$$

•Intermediate indicators

$$M = \max\{|x_i - x_{best}|\} \tag{4}$$

$$x_{new} = 1 - \frac{|x_i - x_{best}|}{M} \tag{5}$$

where $\{x_i\}$ is a set of intermediate indicators sequences, with the best value being x_{best}, and x_{new} being the very large indicator after normalization.

•Interval indicators

$$M = \max\{a - \min\{x_i\}, \max\{x_i\} - b\} \tag{6}$$

$$x_{new} = \begin{cases} 1 - \frac{a - x}{M}, x < a \\ 1, \quad a \le x \le b \\ 1 - \frac{x - b}{M}, x > b \end{cases} \tag{7}$$

where $\{x_i\}$ is a set of intermediate indicators sequences, with the best value being x_{best}, and x_{new} being the very large indicator after normalization.

(3) Data standardization

•Standardization without negative numbers

$$z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} {x_{ij}}^2}} \tag{8}$$

where z_{ij} is the standardized value.

•Standardization with negative numbers

$$z_{ij} = \frac{x_{ij} - \min\{x_{1j}, x_{2j}, \dots, x_{ij}\}}{\max\{x_{1j}, x_{2j}, \dots, x_{ij}\} - \min\{x_{1j}, x_{2j}, \dots, x_{ij}\}} \tag{9}$$

(4) Calculate the proportion of each indicator

Calculate the probability matrix p:

$$p_{ij} = \frac{z_{ij}}{\sum_{i=1}^{n} z_{ij}} \tag{10}$$

Calculate the information entropy e (uncertainty) of each indicator:

$$e_j = -\frac{1}{lnn}\sum_{i=1}^{n} p_{ij}\ln(p_{ij}), j = 1,2, \dots, n \tag{11}$$

(5) Calculate weights

1-e is the information utility value, which is then normalized to obtain the weight.

$$w_j = \frac{1 - e_j}{\sum_{i=1}^{n}(1 - e_j)} \tag{12}$$

(6) Calculate comprehensive performance score

$$S_i = \sum_{j=1}^{n} w_j z_{ij}, i = 1,2, \dots, m \tag{13}$$

In this paper the evaluation indicators are divided into three dimensions: economic(ec), social(so) and environmental(en). Therefore, the combined performance of the indicators for each dimension is defined as

$$S_{i(ec)} = \sum_{j=1}^{n_{(ec)}} w_{j(ec)} z_{ij(ec)}, i = 1,2, \dots, m$$

$$S_{i(so)} = \sum_{j=1}^{n_{(so)}} w_{j(so)} z_{ij(so)}, i = 1,2, \dots, m \tag{14}$$

$$S_{i(en)} = \sum_{j=1}^{n_{(en)}} w_{j(en)} z_{ij(en)}, i = 1,2, \dots, m$$

where $n_{(ec)}$, $n_{(so)}$ and $n_{(en)}$ indicate the number of indicators in three dimensions respectively.

2.4.2 Development Index and Coordination Index

Sustainable development requires not only development, but also the ability to coordinate is essential. In the above formula, $S_{i(ec)}$, $S_{i(so)}$ and $S_{i(en)}$ denote the comprehensive performance scores of the development index in economic, social, and environmental aspects. This paper uses the Development Index (DI) to describe overall development and the Coordination Index (CI) to describe the coherence and coordination of development in the three aspects: economic (ec), social (so) and environmental (en). C_j is now defined as the performance value of the coordination index (urbanization quality) between the economic, social, and environmental dimensions of a sample j. [5]

$$\mathrm{DI} = \mathrm{CF}$$

$$\mathrm{C} = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix}^T, F = [S_{i(ec)} \quad S_{i(so)} \quad S_{i(en)}] \tag{15}$$

where the elements in C give the weights of $S_{i(ec)}$, $S_{i(so)}$ and $S_{i(en)}$.

$$C_j = 1 - \frac{S_j}{\bar{S}} \tag{16}$$

$$S_j = \sqrt{\frac{1}{3}\left[\left(S_{i(ec)} - \bar{S}\right)^2 + \left(S_{i(so)} - \bar{S}\right)^2 + \left(S_{i(en)} - \bar{S}\right)^2\right]} \tag{17}$$

$$\text{where } \bar{S} = \frac{1}{3}(S_{i(ec)} + S_{i(so)} + S_{i(en)})$$

where S_j is the standard deviation of the performance scores of three development index dimensions.

2.5 The Ge-Matrix for Scoring Sustainability

The GE-matrix method, also known as the McKinsey's matrix, nine box matrix method, or industry attractiveness matrix. It is a portfolio analysis method founded by General Electric Company of the United States in the 1970s. Evaluate existing business through two dimensions: the market competitiveness of enterprises and the market attractiveness of the industry. Each dimension is divided into three layers and then nine squares represent combinations of different levels, with each dimension assigned a different color. As Figure 2, the strategies adopted for different color areas are different.

Although there are differences between national development and company operations, there are still similarities from the perspectives of development and attractiveness. For example, the economic benefits of a company are the main indicator of its attractiveness, and the development strength of similar countries also represents its development index. In addition, the coordination index of both reflects their comprehensive strength. Therefore, the GE-matrix is used to the sustainability of countries.

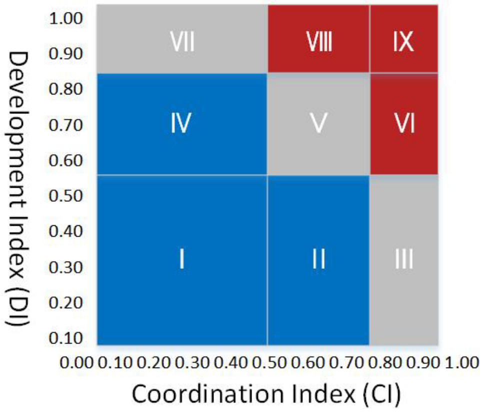

Fig.2 The Ge-Matrix to Evaluate Development Index and Coordination Index

Table 3 Classification of Di and CI

DI	Development level
0-0.5 (I II III)	Weak development
0.5-0.8(IV V VI)	Medium development
0.8-1 (VII VIII IX)	Strong development
CI	Coordination level
0-0.5 (I IV VII)	Weak coordination
0.5-0.8(II V VIII)	Medium coordination
0.8-1 (III VI IX)	Strong coordination

When we determine the CI values x and DI values y for a country and a year, we determine a position (x, y) in the GE-matrix. The distance between (x, y) and the ideal location (1, 1) is considered the country's sustainable development score within one year. Table 3 shows the final score classification results. The higher the score, the stronger a country's sustainable development ability. Based on the entropy GE matrix diagram mentioned above, targeted development strategies for each region can be formulated. For areas with lower levels of development (e.g. I, II, and III), local governments should focus on infrastructure construction and urgently improve urbanization levels. For areas with fast development rate but low coordination ((e.g. VII), the government should optimize the industrial structure and increase the rural urbanization rate. For regions with fast development rate and high coordination ((e.g. VIII and IX), a hopping model is the best choice. For those regions with moderate development rates ((e.g. IV, V, and VI), the government should continue to develop existing strategies and innovate to develop new technology industries, and continue to promote coordinated urban-rural development programs.

2.6 Forecast for Sustainability

Lanchester's Equation

The Lanchester's equation is a set of differential equations that describe the relationship between the changes in the forces of both sides during the process of engagement. Because it was created by F.W. Lanchester, it has its name. The Lanchester's equation can not only predict the combat damage of troops and equipment during the combat process, but also evaluate and optimize combat strategies. The equation is described as:

$$\begin{cases} \dfrac{dx}{dt} = -ay - \alpha x + u(t) \\ \dfrac{dy}{dt} = -bx - \beta y + v(t) \end{cases} \tag{18}$$

Where a and b represent the intensity of the attack on both sides; α and β are troop losses due to reasons other than the attack, respectively; u(t) and v(t) denote the increase in troops.

Forecasting Equation

The Lanchester's equation can observe the dynamic process in warfare and provide the ability to predict changes in sustainability. Therefore, not only can the Lanchester's equation be used to describe the influence of internal factors on one's own side, but also the interaction between various factors and the influence from outside the system. This ability is well suited for national sustainable development assessment and prediction. For example, in the indicator system listed above, life expectancy, education expenditure, health expenditure, etc. reflect the quality of life of citizens in a country, which is also influenced by environmental factors (forest cover, carbon dioxide emissions, etc.) and economic factors (per capita GDP, inflation rate, etc.). The use of the Lanchester's equation is able to capture the interaction between the environmental, social, and economic levels. In addition, there are influences from abroad, such as natural and man-made disasters. Therefore, a new Lanchester's equation can be obtained:

$$\begin{cases} \frac{dS_{i(ec)}}{dt} = \alpha_{11}S_{i(ec)}(t) + \alpha_{12}S_{i(so)}(t) + \alpha_{13}S_{i(en)}(t) + \beta_1\widehat{U_1}(t) \\ \frac{dS_{i(so)}}{dt} = \alpha_{21}S_{i(ec)}(t) + \alpha_{22}S_{i(so)}(t) + \alpha_{23}S_{i(en)}(t) + \beta_2\widehat{U_2}(t) \\ \frac{dS_{i(en)}}{dt} = \alpha_{31}S_{i(ec)}(t) + \alpha_{32}S_{i(so)}(t) + \alpha_{33}S_{i(en)}(t) + \beta_3\widehat{U_3}(t) \end{cases} \tag{19}$$

where $\widehat{U_\iota}(t)$ indicate the country sustainability.

Using denotation ways from state space theory, we get the state equation:

$$\frac{dS}{dt} = AS + B\widehat{U} \tag{20}$$

$$t = kT \quad k = 1, 2, 3, \dots n \tag{21}$$

where $A = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix}, S = \begin{bmatrix} S_{i(ec)} \\ S_{i(so)} \\ S_{i(en)} \end{bmatrix}, B = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}, \widehat{U} = \begin{bmatrix} \widehat{U_1} \\ \widehat{U_2} \\ \widehat{U_3} \end{bmatrix}^T$.

Among them, S represents the comprehensive performance of a country's sustainability mentioned above; A describes the mutual influence between the three levels; B is a parameter that controls external influences; $\widehat{U}$ expresses the influence from abroad. Considering practical factors, the dynamic changes in future sustainability and potential future value of comprehensive performance S, continuous time t, are transformed from discrete time T.

3. Model Improvement

The sustainability of a country is not only determined by internal factors, but may also be influenced by external uncertainties. For example, globalization and assistance from friendly countries have a positive effect on sustainability; The spread of major diseases and natural disasters have negative impacts. In equation(17), matrix $B\widehat{U}$ describes the external effects. Then we let

$$B\widehat{U} = \hat{A}F \tag{22}$$

where $\hat{A}$ is a perturbation matrix of *F*. Since $B\widehat{U}$ is commonly respected as a random matrix whose elements obey normal distribution, where μ=0 and σ determined by the degree of effect. Elements of $\hat{A}$ are $\hat{\alpha}_{ij}$ which are perturbation increments and $\hat{\alpha}_{ij} \sim N(0, \sigma)$. Therefore, the final expression of the equation of state is:

$$\frac{dS}{dt} = (G_{cA}A + G_{bA} + \hat{A})S \tag{23}$$

$$G_A = G_{cA}A + G_{bA} \tag{24}$$

where $G_{cA} = \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix}, G_{bA} = \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix}$.

G_{cA} is an indirect control matrix used to control system matrix A, while G_{bA} is a direct control matrix that affects the overall score directly. c_i is the indirect control element in the plan whose properties are shown in Table 4.

Table 4 Effects of Lan According to c_i and $\hat{\alpha}_{ij}$

	$c_i < 1$	$c_i > 1$
$\hat{\alpha}_{ij} < 1$	Plan decreases original negative effect	Plan increases original negative effect
$\hat{\alpha}_{ij} > 1$	Plan decreases original positive effect	Plan increases original positive effect

4. A Case Study on Congo

By using the data collected from the World Bank Database[6] and setting σ = 0.4, a revised forecast on Congo can be obtained. The modified control elements are shown in Table 5.

Table 5 Modified Control Elements of the Next Decade

Control Elements	2024~2028	2029~2033
c_1	0.05	0.05
c_2	0.05	0.1
c_3	0.4	0.35
b_1	-0.04	-0.02
b_2	0	-0.03
b_3	0.5	0.45

When σ = 0.4, the overall performance growth trend is good, which proves that the development plan in Congo is still favorable. However, external factors have increased the difficulty of prediction, and the Congo development plan can only have external impacts to a certain extent.

5. Conclusion

Dividing indicators into three levels, each containing multiple indicators, and using recognized and statistically significant indicators as much as possible will provide more accurate information and better evaluate a country's sustainability. The use of Development Index (DI) and Collaboration Index (CI) to represent a country's development index and coordination index is in line with practical significance, without bias, making it easier for leaders to make the right decisions. The policy can be observed through the Lanchester's equation during implementation, avoiding situations of loss of control. Use a control matrix to represent the control factors of policies, where changes in the control matrix are equivalent to aid and policies, clear and detailed. In addition to the indicators already included in this model, there are often many factors that are difficult to quantify, such as climate factors and food security issues. And the composition of certain factors often has multiple reasons, and cannot be accurately described by just one index, which may lead to amplifying or narrowing the impact of one factor on sustainability. In the model, only the linear part of the exponential relationship is roughly calculated, which can lead to errors. Therefore, it is necessary to conduct more comprehensive quantitative and qualitative analysis, add nonlinear relationships and accurately estimate missing parameters in the model, evaluate how to weight individual indicators to reflect their importance, and evaluate the unsustainable threshold for each category. Make the direct connection between specific strategies and sustainability performance more closely linked. Collect more research data on a broader scale to ensure the accuracy of the model.

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